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Brief Communication

A re-examination of the data of Lockhart–Martinelli

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1. Introduction

Lockhart and Martinelli (1949), hereinafter referred to as LM, presented an empirical correlation for the frictional pressure drop in liquid–gas two phase flow in horizontal pipes. LM recognizes four categories of isothermal two-phase, two-component flows based on whether each phase is flowing in the viscous (v) or turbulent (t) flow regime.

LM used data for the simultaneous flow of air and various liquids including benzene, kerosene, water and various oils in pipes varying in diameter from 0.0586 in. to 1.017 in. Having divided the available data into the four flow categories of liquid–gas t–t, t–v, v–t and v–v flows, plots of ϕ vs X were made for each flow type. ϕ is the square root of the ratio of the two phase frictional pressure drop to the superficial single phase pressure drop, with the subscript L or G signifying whether the single phase pressure drop used referred to the liquid phase or gas phase flowing alone. X is the square root of the ratio of the pressure drop in the pipe if the liquid flowed alone to the pressure drop if the gas flowed alone. However, these four flow categories are based on the superficial parameters of the individual single phase flows and are therefore unrelated to the forces that occur which depend on the actual flow pattern of the flowing two phase mixture.

Mandhane et al. (1974) developed a flow pattern map (hereinafter referred to as the Mandhane–Gregory–Aziz map) which divides two phase flows into six different flow patterns: annular flow, slug flow, bubbly flow, wavy flow, stratified flow and dispersed flow. The data were plotted in terms of superficial velocities of each phase and both axes are in the logarithmic scale. Taitel and Dukler (1976) presented models based on physical concepts for determining flow regime transitions. Using these mechanistic models, the authors also gave a generalized flow pattern map. The generalized map may be used to calculate the superficial velocities of each of the phases at flow pattern transitions for ease of comparison with the Mandhane–Gregory–Aziz map. It was found that the theoretically based Taitel and Dukler (1976) map was in good agreement with experimental data and the Mandhane–Gregory–Aziz map. In this paper, the LM original data will be re-categorized based on the Mandhane–Gregory–Aziz map.

2. Re-examining the original data of Lockhart–Martinelli

The data used by LM consisted of experimental results obtained from a number of sources as detailed in the original paper and covered 810 data sets including 191 data sets that are for inclined and vertical pipes. It should be noted that only the 619 data sets for horizontal flow are used in this paper.

LM divided the data into the four flow mechanisms of liquid–gas t–t, t–v, v–t and v–v flow, and plots of ϕ vs X were made for each flow type. Empirical correlation curves for each flow mechanism (t–t t–v v–t and v–v) were developed and the coordinates of each correlation curve were given in the LM paper.

In this paper, the original data are recalculated following the procedures of LM. Once the data are separated into the four flow mechanisms based on the superficial Reynolds number of the gas phase and liquid phase respectively, the corresponding values of X, ϕ_G and ϕ_L are calculated, and the data points are plotted on the ϕ_G–X diagram. These data points are compared with the four LM correlation curves respectively. It is noted that LM only presented the graphs for ϕ_G vs X for the t–t, v–t and v–v flow mechanisms, and the graph of t–v flow was not given.

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It should be noted that the correlation curves in Figs. 1–4 are based on the coordinates provided in the paper by LM. The variable ϕ_L is directly related to ϕ_G and is therefore not plotted in these figures. Although four types of correlation curves are given by LM, there is a significant difference in the number of data points associated with each of the correlation curves. There is a significant number of data points in the t–t and v–t categories (refer Figs. 1 and 3, Table 1) to support the development of the respective correlation curves, while the t–v curve (Fig. 2) is only based on a small number of data points in the range of $X > 10$. Furthermore, for each of the flow mechanisms, the data points do not cover the full range of the correlation. The majority of data shown in Figs. 1, 3 and 4 are located in the range of $10^{-1} < X < 2 \times 10^{1}$ and there are very few data points for low values of X.

In the original LM paper, there is no mention of how the correlation curves are developed from the data points and there is also no evidence of any statistical analysis. It appears that the curves were drawn by following the general trend of the data points. Fur-

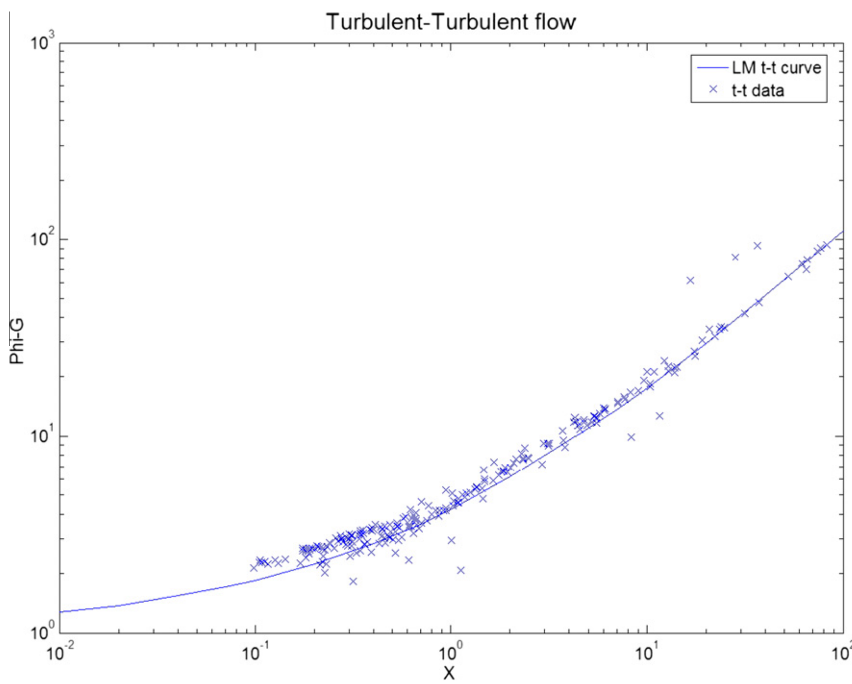

Fig. 1. The data and correlation curve for turbulent–turbulent (t–t) flow.

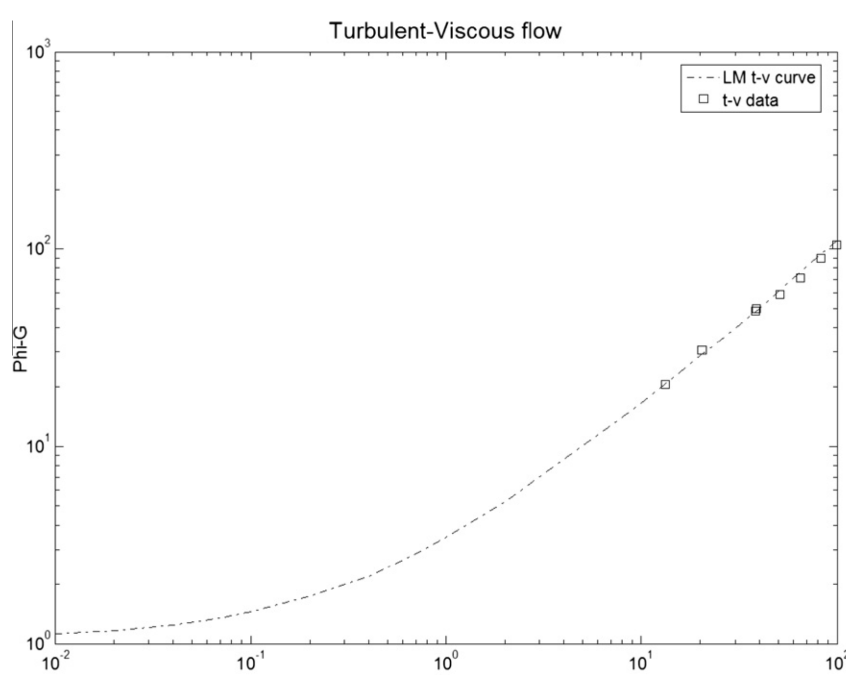

Fig. 2. The data and correlation curve for turbulent–viscous (t–v) flow.

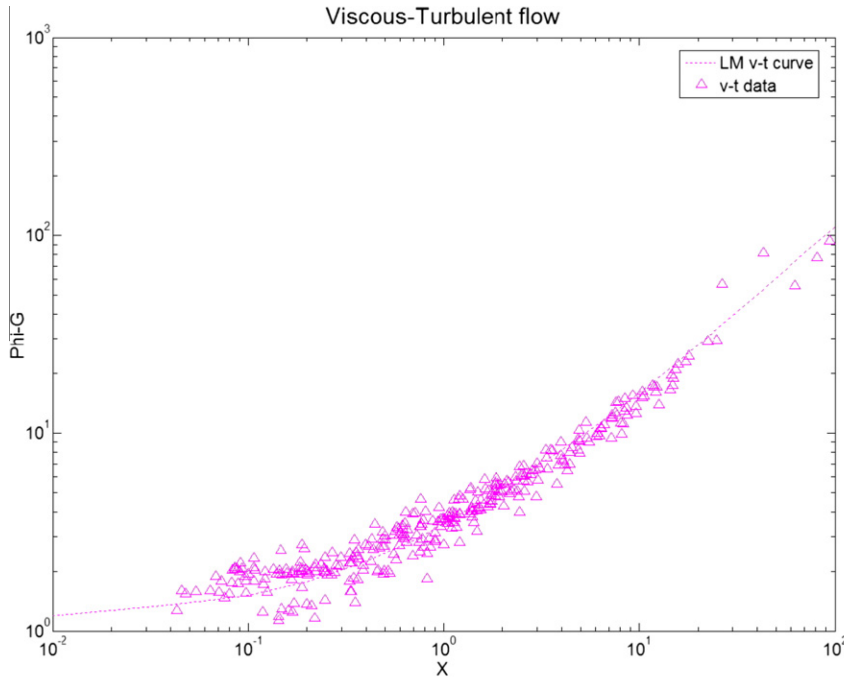

Fig. 3. The data and correlation curve for viscous–turbulent (v–t) flow.

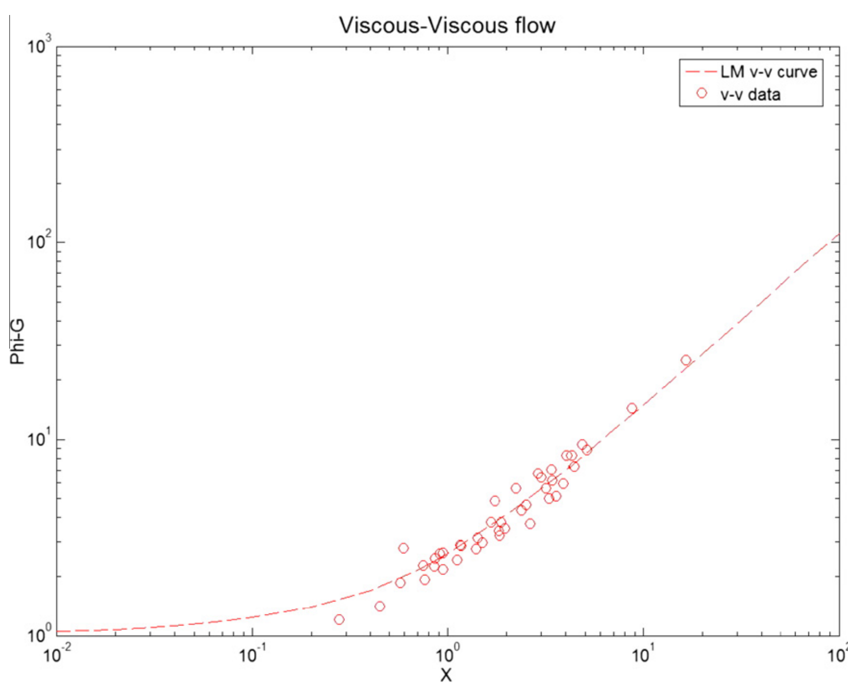

Fig. 4. The data and correlation curve for viscous–viscous (v–v) flow.

thermore, from the original graph of the correlation curves given in LM, it is noted that the middle and some of the right-hand portions of the curves are shown as "solid lines" while the left-hand portion of the curves are drawn as "dashed lines". It is clear that the "dashed lines" are not supported by data points and are extrapolations.

When the LM paper was published, computers and numerical analysis were not so readily accessible. With the help of modern computers, the goodness of fit of data to empirical correlations can be analyzed and new empirical curves that better fit the existing data points may be obtained through the non-linear least squares method. The relationship between the four types of data

Table 1
Accuracy of the LM curves compared with the original data used.

	Number of points	Mean absolute percentage error (%)
t–t curve	229	13.4
t–v curve	9	3.5
v–t curve	339	14.3
v–v curve	42	12.0

points and their corresponding curves are analyzed and tabulated in Table 1.

In Table 1, "Mean Absolute Percentage Error" refers to the vertical distance between the data point and the curve expressed as a percentage deviation from the curve. It is obvious the t–v curve has a percentage error significantly lower than for the other curves. However, this does not necessarily mean that the t–v curve is the best-fitted correlation because there are only nine data points associated with this curve. In addition, these data points are in a very narrow range of $10^1 < X < 10^2$ as shown in Fig. 2 while the

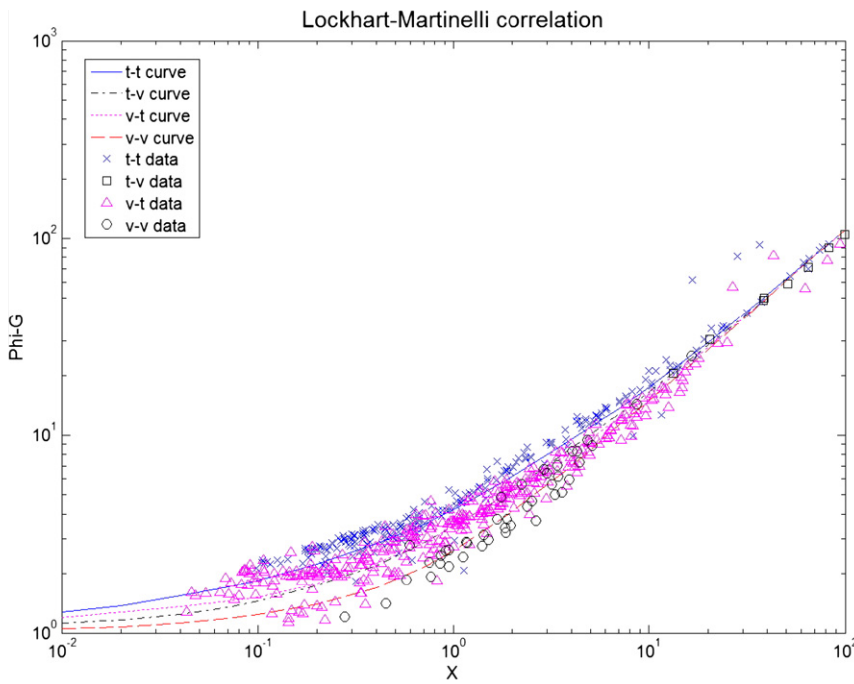

Fig. 5. Plots of the four flow mechanisms (t–t, t–v, v–t, and v–v).

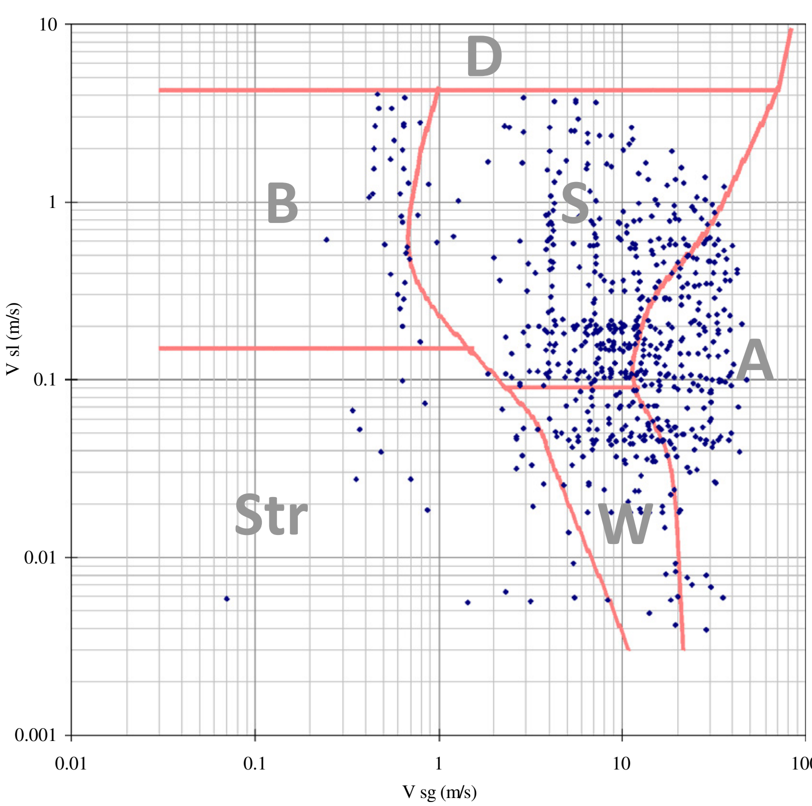

Fig. 6. The 619 data points of LM plotted on the Mandhane–Gregory–Aziz map (A = Annular flow, S = Slug flow, W = Wavy flow, B = Bubbly flow, Str = Stratified flow, D = Dispersed flow).

Table 2
Data of LM categorized based on flow pattern.

	Number of data	Percentage of data (%)
Annular flow	191	30.9
Slug flow	277	44.7
Wavy flow	94	15.2
Bubbly flow	32	5.2
Stratified flow	25	4.0
Dispersed flow	0	0.0
Total	619	100.0

empirical correlation given was for the range $10^{-2} < X < 10^{2}$. The t–t, v–t and v–v curves have similar but larger values of percentage error compared with the t–v curve. To facilitate visual comparison, all the four curves and their corresponding data points are plotted on the same chart as shown in Fig. 5.

In Fig. 5 it can be observed that most of the t–t data points lie above the v–t data points while the v–v points tend to be lower. The relative position of each group of data points is reflected by the correlation curves: the t–t curve lies at the top, the v–v curve lies at the bottom, the t–v curve and the v–t curves are close together and lie between the t–t and the v–v curves. Based on Fig. 5, it can be concluded that the LM t–t, v–t and v–v reflect reasonably well the available data. Fig. 5 contains a number of outliers. Some of these outliers are in the "transition region" with the Reynolds number of either phase fall between 1000 and 2000. It is noted that these data in the "transition region" were not shown in the original LM paper.

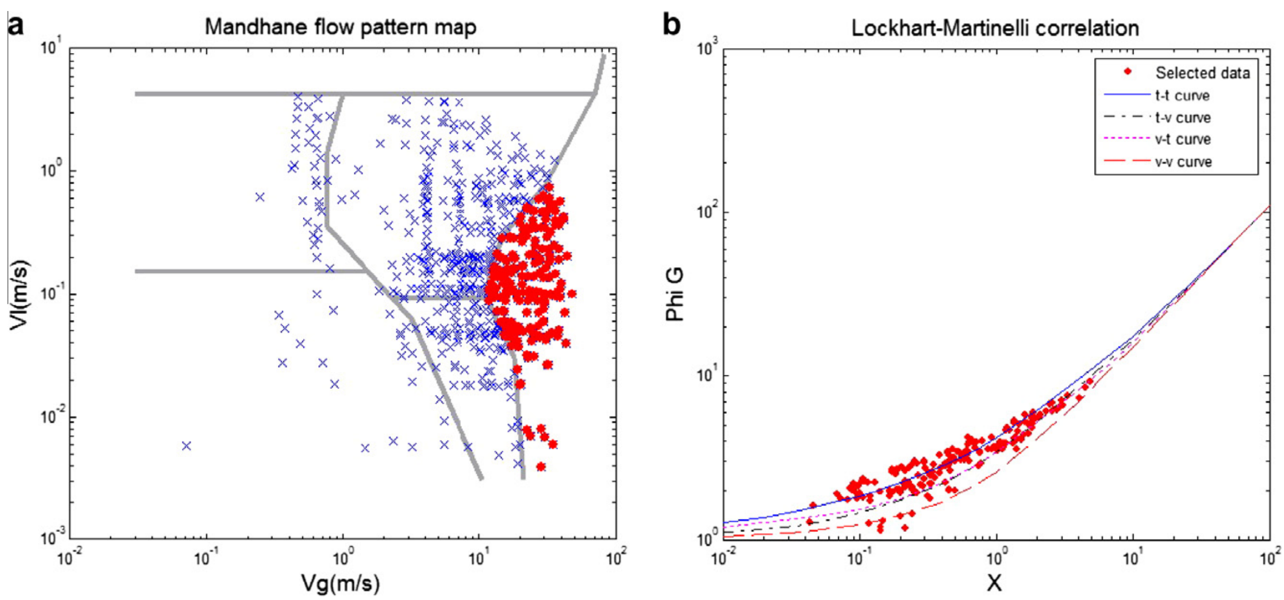

Fig. 7a and b. Annular flow data on Mandhane–Gregory–Aziz map and the ϕ–X plot.

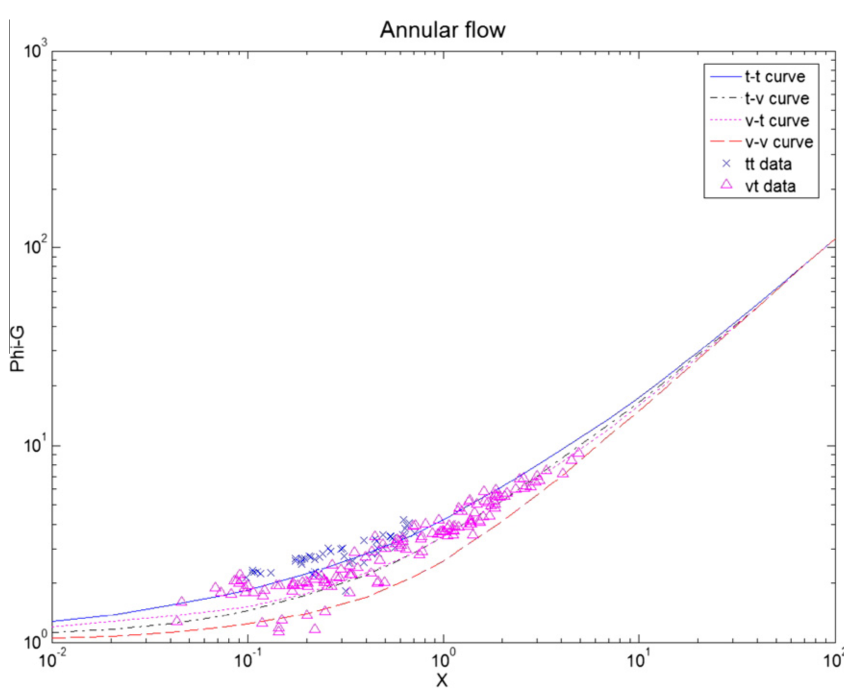

Fig. 7c. Annular flow data separated into different categories (t–t and v–t).

3. Re-categorizing data according to flow pattern

The original LM data contain no information on flow patterns. In order to re-categorize the data according to flow pattern, we need to make use of a suitable and reliable existing flow pattern map. As discussed before, two flow pattern maps, the Mandhane–Gregory–Aziz map and the Taitel–Dukler map, are generally accepted as reliable.

Having calculated the superficial velocities of each of the phases, the data are plotted on the Mandhane–Gregory–Aziz flow pattern map as scatter points in Fig. 6.

Fig. 6 shows that the data used by LM fall into five categories in terms of flow patterns: A, Annular flow; B, Bubbly flow; W, Wave flow; S, Slug flow and Str, Stratified flow. There are no data in the D, Dispersed flow region. The X-axis "V_{sg}" is the superficial velocity of the gas phase, while the Y-axis "V_{sl}" is the superficial velocity of the liquid phase. It can be observed that the majority of the data fall within the annular, slug and wavy flow patterns. A few points fall within the stratified flow and the bubbly flow patterns. The number of points in each flow pattern and their percentage are shown in Table 2.

The data within each flow pattern category are plotted separately in terms of ϕ_G vs X as shown in Fig. 7 (Annular), Fig. 8 (Slug), Fig. 9 (Wavy), Fig. 10 (Bubbly) and Fig. 11 (Stratified). In Figs. 7a, 8a, 9a, 10a and 11a, the data points in each selected flow pattern category are plotted as "asterisks" while the remaining data points

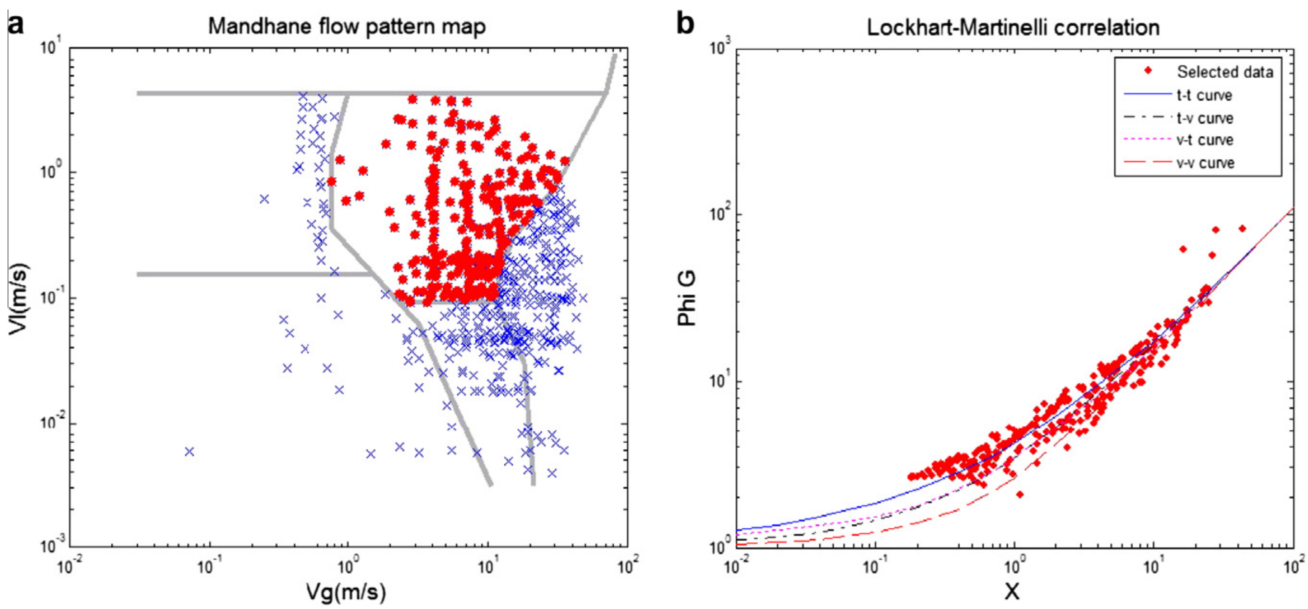

Fig. 8a and b. Slug flow data on Mandhane–Gregory–Aziz map and the ϕ–X plot.

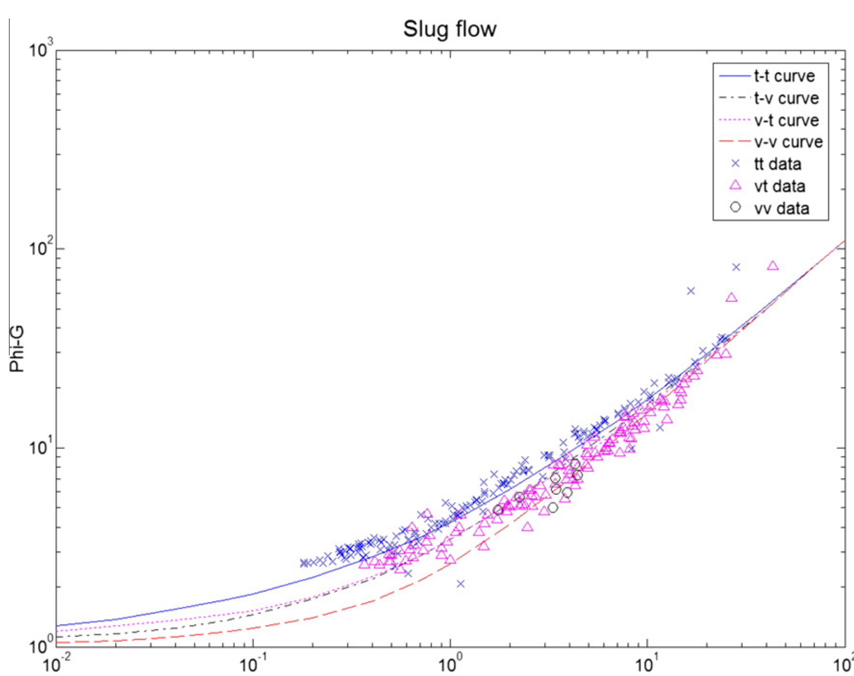

Fig. 8c. Slug flow data separated into different categories (t–t, v–t and v–v).

are plotted as "crosses". The selected "asterisks" are plotted as ϕ_G vs. X in the corresponding Figs. 7b, 8b, 9b, 10b and 11b, and the data points in each flow pattern are further shown in terms of the flow mechanisms of t–t, t–v, v–t and v–v in Figs. 7c, 8c, 9c, 10c and 11c. For example, in Fig. 7a, the annular flow data are shown as "asterisks" while the ϕ vs X plot is shown in Fig. 7b. Fig. 7c is the enlarged plot of Fig. 7b with the data points further separated into the different flow mechanisms of t–t, t–v, v–t and v–v and they are differentiated by using different symbols. It is noted that in the annular flow data, the data points are confined to the t–t and v–t flow mechanisms.

In each flow pattern, the distribution of data points based on the four flow mechanisms of t–t, t–v, v–t and v–v flow is tabulated in Table 3.

3.1. Annular flow

The annular flow data points fall within the range of $0.05 < X < 5$. As shown in Fig. 7b, these points show considerable scatter for smaller values of X and began to merge at about $X = 1$. The data consist of 50 t–t points and 141 v–t points, no t–v and v–v points. Thus in all of the LM data in this flow pattern the gas

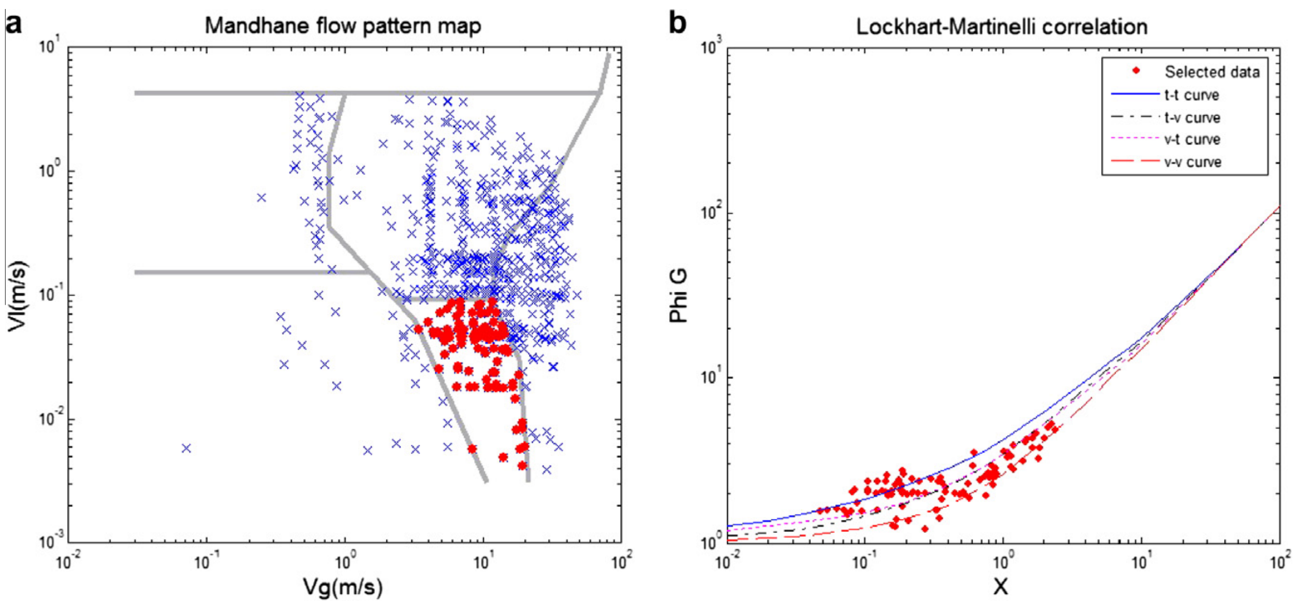

Fig. 9a and b. Wave flow data on Mandhane–Gregory–Aziz map and the ϕ–X plot.

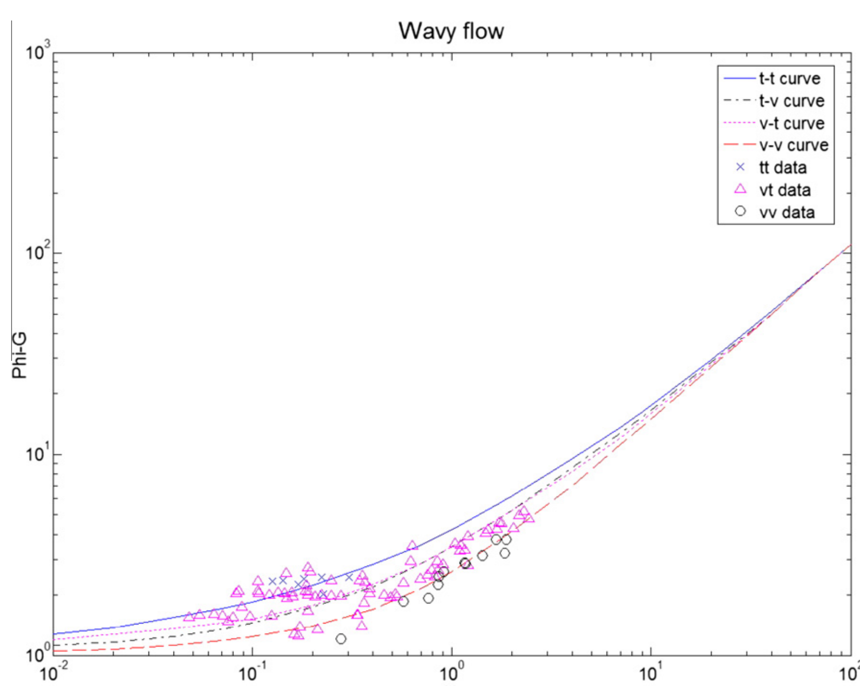

Fig. 9c. Wavy flow data separated into different categories (t–t, v–t and v–v).

phase is always turbulent while the liquid phase is either viscous or turbulent. The annular flow data have values of ϕ_G ranging from slightly above unity to about 10.

3.2. Slug flow

With reference to Fig. 8c, the slug flow data points fall within the range of $0.18 < X < 43.3$ and $2 < \phi_G < 82$ (if the four outliers at the top were excluded, the range becomes $0.18 < X < 25$ and $2 < \phi_G < 36$). The slug flow data contain 155 t–t points, 114 v–t points, 8 v–v points and no t–v point. Close observation of Fig. 8c shows that the t–t data and the v–t data appear to cluster about different curves. Approximately two-thirds of all the t–t points in the LM data bank fall within the slug flow region. In the original LM paper, it was postulated that their correlation required that the flow pattern did not change along the tube length, and thus ''slug'' flow, in which alternate slugs of liquid and gas move down the tube, was eliminated from consideration.

3.3. Wavy flow

In the ϕ_G–X plot given in Fig. 9c for wavy flow, the v–t data covers approximately the range $0.05 < X < 2.5$ while the t–t data is found at about $X < 0.3$ and the v–v data at $X > 0.3$. The highest X va-

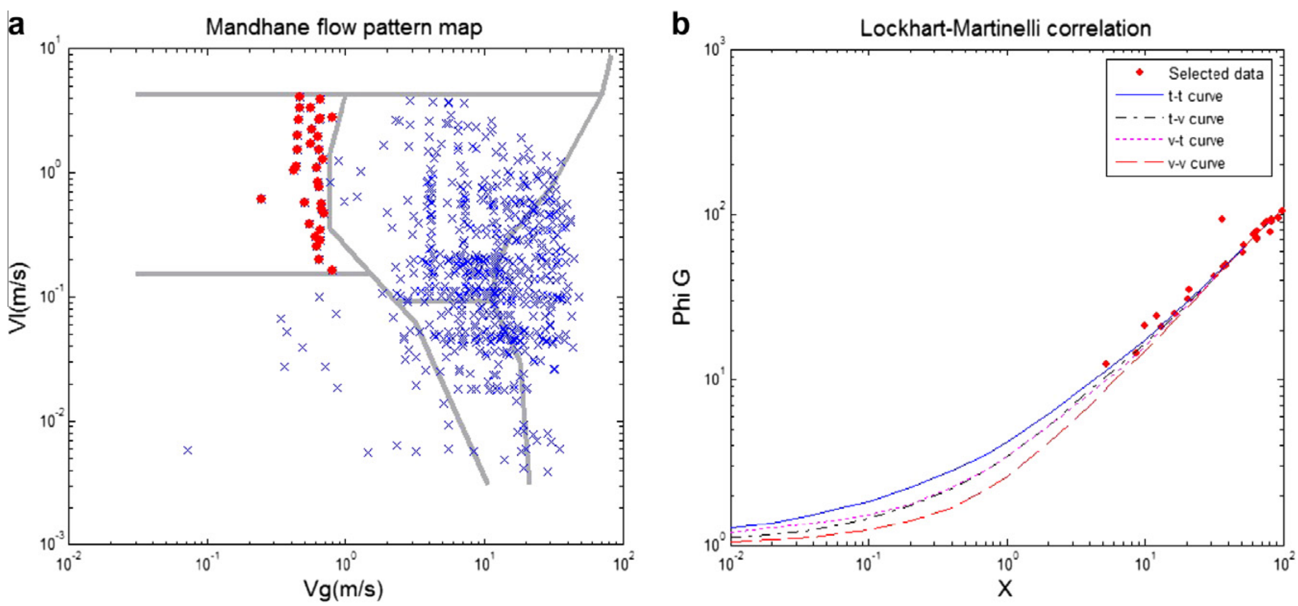

Fig. 10a and b. Bubbly flow data on Mandhane–Gregory–Aziz map and the ϕ–X plot.

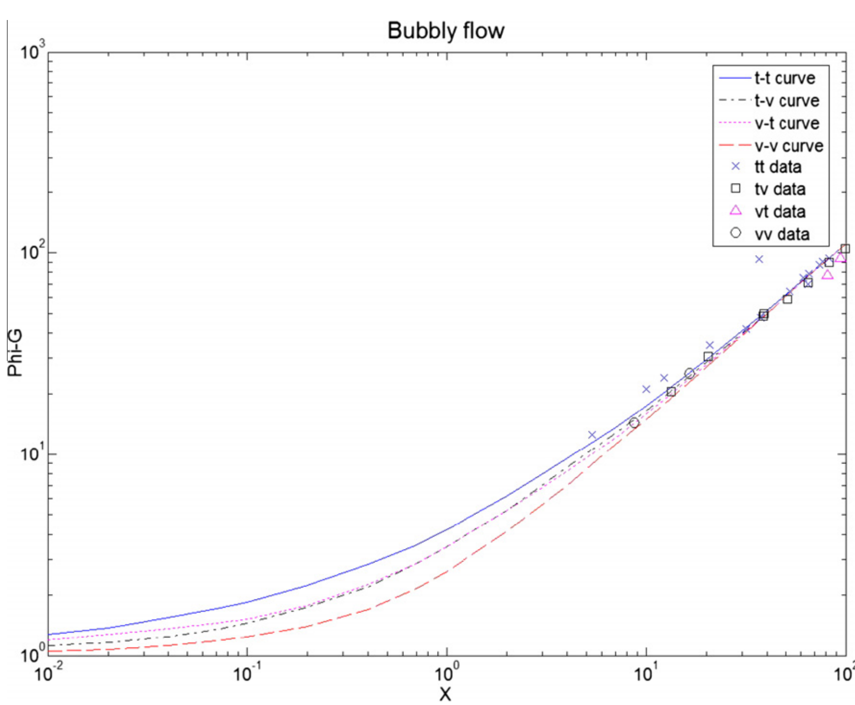

Fig. 10c. Bubbly flow data separated into different categories (t–t, t–v, v–t and v–v).

lue for wavy flow in the data of LM is 2.5 which should be compared with 5 in the case of annular flow. The wavy flow data points overlap somewhat with the annular data points when $X < 0.5$, but they have lower values of ϕ_G than annular flow as X exceeds 0.5. The majority of wavy flow data is made up of the v–t flow mechanism.

3.4. Bubbly flow

As shown in Fig. 10b, the bubbly points only cover a small range of $5 < X < 100$ but they constitute most of the high X value data in LM. These bubbly points show a high degree of agreement with the LM correlations as the four curves merge at values of X close to 100. The bubbly flow covers all flow mechanisms including 16 t–t points, 9 t–v points, 5 v–t points and 2 v–v points. It should be noted that there are only 9 t–v flow data sets in the entire data bank of LM and all these 9 t–v points fall in the bubbly flow region.

3.5. Stratified flow

Stratified points are located within the range $0.3 < X < 11$. The majority of the stratified points belong to the v–v flow mechanism and these points also appear to scatter about the LM v–v curve. It is noted that in the "Discussion" by Gazley and Bergelin that accompanied the original LM paper, it was pointed out that the LM

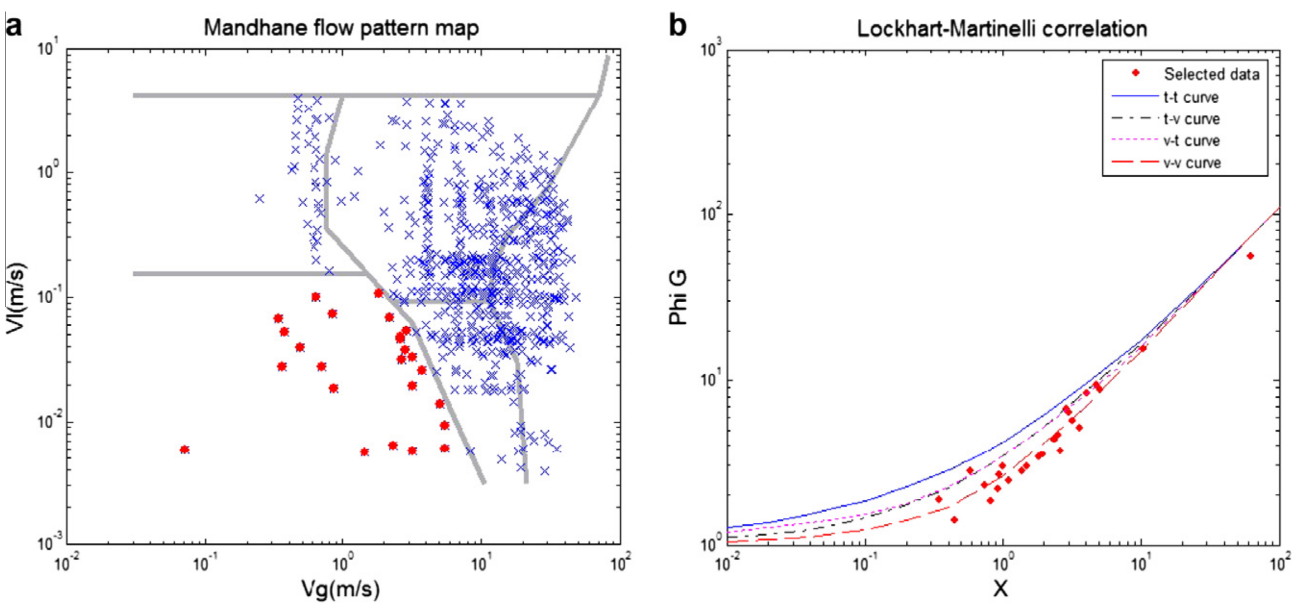

Fig. 11a and b. Stratified flow data on Mandhane–Gregory–Aziz map and the ϕ–X plot.

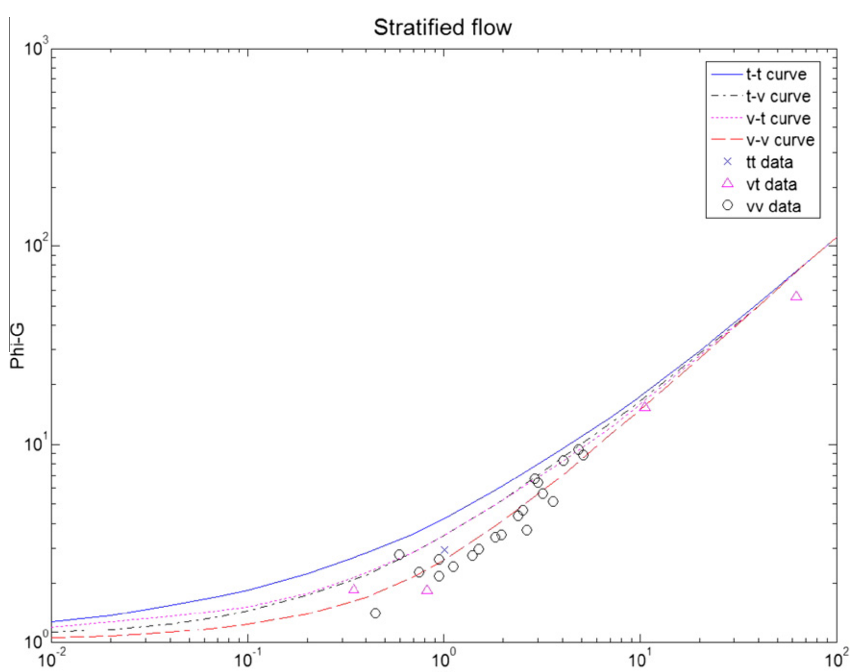

Fig. 11c. Stratified flow data separated into different categories (t–t, v–t and v–v).

correlation was not valid for stratified flow. 25 data points for stratified flow were included in the LM correlation.

After all the data have been re-categorized according to flow pattern, the new data groups are compared with the LM curves. Again, the "Mean Absolute Percentage Error" is used for making the comparison and the results are tabulated in Table 4. It can be seen that the t–t curve is the best correlation for the annular (13.4% error), bubbly (9.0% error) and slug (15.8% error) data used by LM. The wavy data showed an error greater than about 20% when compared with any one of the LM curves, while the stratified data is best represented by the v–v curve with an average error of 14.3%. Thus, when the data were categorized according to flow patterns, none of the four curves (t–t, t–v, v–t and v–v) provided improved correlation, but with the exception of the bubbly flow data which showed an averaged error of 9.7%. It should be noted that the bubbly data points are located at large values of X where the four ϕ_G–X curves tend to merge.

To facilitate visual comparison, all the new data groups based on the flow pattern are plotted on the same ϕ_G–X chart in Fig. 12. It can be seen that the annular, slug and wavy data appear to overlap with each other to some extent. The bubbly and stratified data are small in number, but they can be easily distinguished from the rest, because the bubbly data have large X values and the stratified data deviate negatively from the LM curves.

Table 3
Breakdown of LM data according to flow patterns and flow mechanisms.

	Annular	Slug	Wavy	Bubbly	Stratified	Total
t–t flow	50	155	7	16	1	229
t–v flow	0	0	0	9	0	9
v–t flow	141	114	75	5	4	339
v–v flow	0	8	12	2	20	42
Total	191	277	94	32	25	619

Table 4
The Absolute Mean Percentage Errors compared to the LM correlation based on flow patterns and flow mechanisms.

	t–t curve (%)	t–v curve (%)	v–t curve (%)	v–v curve (%)
Flow patterns				
Annular data (191)	13.4	22.7	20.8	50.6
Bubbly data (32)	9.0	9.7	10.6	12.2
Slug data (277)	15.8	24.2	23.9	45.1
Wavy data (94)	22.1	21.6	19.8	31.5
Stratified data (25)	33.4	33.4	22.2	14.3
Flow mechanisms				
t–t data (229)	13.4	–	–	–
t–v data (9)	–	3.5	–	–
v–t data (339)	–	–	14.3	–
v–v data (42)	–	–	–	12.0

4. Conclusion

In this paper, the original data of LM are re-analyzed. It is shown that in the original data bank, for horizontal flow, there are 229 data points in the t–t flow mechanism, 9 in t–v, 339 in v–t and 42 in v–v. The data points employed to develop the t–v correlation only cover a small range of the curve and are all in the bubbly flow pattern, and most part of the t–v curve is plotted without the support of data points. In the original LM categorizations, the errors were found to be 13.4% for t–t flow (229 data points), 3.5% for t–v flow (nine data points), 14.3% for v–t flow (339 data points) and 12.0% for v–v flow (42 data points).

The original data have been re-categorized based on the flow pattern using the Mandhane–Gregory–Aziz flow pattern map. Within each of the Annular, Bubbly, Slug, Wavy and Stratified data groups, comparisons were made against the LM t–t, t–v, v–t and v–v curves. It is noted that there are no data in the Dispersed flow region. Except for the Bubbly data, the deviations in each case are greater than the deviations based on the original LM data groupings of t–t, t–v, v–t and v–v. Although data groupings based on flow pattern show greater scatter compared with the original LM groupings, the behaviour of the flow pattern based groupings still exhibit a general trend when plotted on the ϕ_G–X coordinates and this may form the basis for the development of flow pattern based correlations.

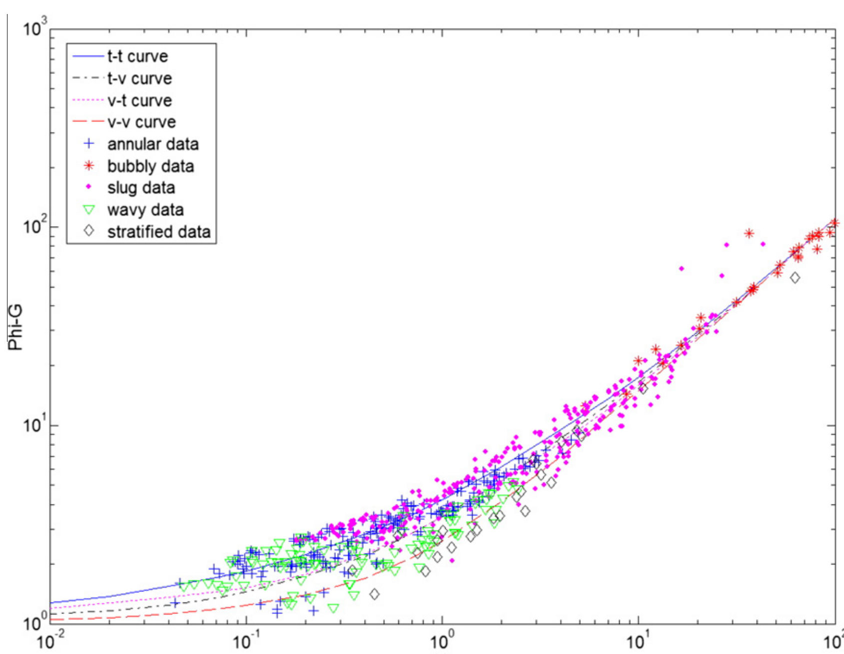

Fig. 12. The ϕ_G–X plot of all flow patterns compared with LM curves.

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